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# Dom-Total Chromatic Number of Thorny Graph

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**ABSTRACT:** Let  $G$  be a graph. For a given  $\chi$ -total colouring of a graph  $G$ , a dominating set  $S \subseteq V(G)$  is said to be a dom-total colouring set if it contains atleast one vertex of each colour class of  $G$ . The dom-total chromatic number of a graph  $G$  is the minimal cardinality taken over all its dom-total colouring sets and is denoted by  $\gamma_{d-tc}(G)$ . In this paper, we introduce algorithms to obtain the dom- total colouring and dom-total chromatic number thorny graphs.

**KEYWORDS:** Chromatic number, Dominating set, Domination number, Dom-colouring set, Dom-chromatic number, Dom- total colouring set, Dom-total chromatic number.

## I. INTRODUCTION

All graphs considered in this paper are finite, undirected graphs and we follow standard definition of graph theory as found in [1]. Let  $G = (V, E)$  be a graph. The open neighborhood  $N(v)$  of a vertex  $v \in V(G)$  consists of the set of all vertices adjacent to  $v$ . The closed neighborhood of  $v$  is  $N[v] = N(v) \cup \{v\}$ . An induced subgraph  $G[S]$ , where  $S$  of a graph  $G$  is a graph formed from a subset  $S$  of the vertices of  $G$  and all of the edges connecting pairs of vertices in  $S$ . A graph in which every pair of vertices is joined by exactly one edge is called complete graph. A complete bi partite graph is a graph whose vertices can be partitioned into two subsets  $V_1$  and  $V_2$  such that no edge has both end points in the same subset, and each vertex of  $V_1$  is connected to every vertex of  $V_2$  and vice -verse. A star graph  $S_n$  is the complete bipartite graph  $K_{1,n-1}$  (A tree with one internal node and  $n-1$  leaves).

The path and cycle of order  $n$  are denoted by  $P_n$  and  $C_n$  respectively. For any two graphs  $G$  and  $H$ , we define the cartesian product, denoted by  $G \times H$ , to be the graph with vertex set  $V(G) \times V(H)$  and edges between two vertices  $(u_1, v_1)$  and  $(u_2, v_2)$  iff either  $u_1 = u_2$  and  $v_1 v_2 \in E(H)$  or  $u_1 u_2 \in E(G)$  and  $v_1 = v_2$ .

A subset  $S$  of  $V$  is called a dominating set if every vertex in  $V-S$  is adjacent to atleast one vertex in  $S$ . The dominating set is minimal dominating set if no proper subset of  $S$  is a dominating set of  $G$ . The domination number  $\gamma$  is the minimum cardinality taken over all minimal dominating set of  $G$ . A  $\gamma$ -set is any minimal dominating set with cardinality  $\gamma$ .

A proper coloring of  $G$  is an assignment of colors to the vertices of  $G$  such that adjacent vertices have different colors. The minimum number of colors for which there exists a proper coloring of  $G$  is called chromatic number of  $G$  and is denoted by  $\chi(G)$ . A dominator coloring of  $G$  is a proper coloring of  $G$  in which every vertex of  $G$  dominates atleast one color class. The dominator chromatic number is denoted by  $\chi_d(G)$  and is defined by the minimum number of colors needed in a dominator coloring of  $G$ . This concept was introduced by Raluca Michelle Gera in 2006[2].

In a proper coloring  $C$  of  $G$ , a color class of  $C$  is a set consisting of all those vertices assigned the same color. Let  $C^l$  be a minimal dominator coloring of  $G$ . We say that a color class  $c_i \in C^l$  is called a non-dominated color class (n-d color class) if it is not dominated by any vertex of  $G$ . These color classes are also called repeated color classes. For a given  $\chi$ -total colouring of a graph  $G$ , a dominating set  $S \subseteq V(G)$  is said to be a dom- total colouring set if it contains atleast one vertex of each colour class of  $G$ . The dom-total chromatic number of a graph  $G$  is the minimal cardinality taken over all its dom-total colouring sets and is denoted by  $\gamma_{d-tc}(G)$ . Let  $G$  be an undirected graph with no loops and parallel edges. The graph formed by subdividing each edge exactly once and joining all the non -adjacent vertices of the graph is called the central graph. It is denoted by the symbol  $C(G)$ .

Let  $G = (V, E)$  be a graph. The thorny graph of  $G$  is denoted by  $G^*$ , is the graph obtained by attaching  $r \geq 0$  new pendant vertices to each vertex of  $G$ .

In other words, for every vertex  $v \in V(G)$ ,  $r$  new vertices of degree one are added and each of them is joined to  $v$  by an edge. These newly added vertices are called thorns or pendant vertices.

## II. MAIN RESULTS

### Theorem 1

The Dom-Total chromatic number of thorny graph  $G^*$  of  $G$  is  $\gamma_{d-tc}(G^*) = \Delta(G^*) + 1 - n$

#### Proof:

Let  $G^*$  be the thorny graph obtained from  $G$  by attaching  $r \geq 0$  pendant vertices to each vertex of  $G$ . Each pendant vertex contributes one additional edge incident to the corresponding vertex of  $G$ .

Hence, if a vertex  $v \in V(G)$  has degree  $d_G(v)$ , then its degree in  $G^*$  become  $d_{G^*}(v) = d_G(v) + r$ .

Therefore, the maximum degree of the thorny graph is  $\Delta(G^*) = \Delta(G) + r$ .

In the total coloring, all edges incident to a vertex of maximum degree and the vertex itself must receive distinct colors.

Therefore, the total chromatic number of thorny graph is at least  $\Delta(G^*) + 1$

i.e.,  $\chi_T(G^*) \geq \Delta(G^*) + 1$

To dominate all the vertices of  $G^*$ , each vertex  $v \in V(G)$  dominates its attached pendant vertices(thorns) and itself.

Hence, the minimum domination number of  $G^*$  is  $\gamma(G^*) = |V(G)|$

Let  $|V(G)| = n$  then  $\gamma(G^*) = n$

We have  $\gamma_{d-tc}(G^*) = \chi_T(G^*) - \gamma(G^*) = \Delta(G^*) + 1 - n$

Hence, the Dom-Total chromatic number of Thorny graph is  $\gamma_{d-tc}(G^*) = \Delta(G^*) + 1 - n$

### Theorem 2

The Dom-Total chromatic number of thorny path  $P_n^*$  is  $\gamma_{d-tc}(P_n^*) = r + 3 - n, \forall n \geq 3$

#### Proof:

In the Path graph  $P_n$ , the maximum degree is  $\Delta(P_n) = 2$

In the thorny path graph  $P_n^*$ , each vertex is attached with  $r$  pendant vertices. Hence, the maximum degree becomes  $\Delta(P_n^*) = r + 2$  for  $n \geq 3$

Since the vertex itself must receive a color different from all its incident edges, one additional color is required. So, the total chromatic number of thorny path graph is

$\chi_T(P_n^*) = r + 2 + 1 = r + 3 \forall n \geq 3$ .

To dominate all the vertices of  $P_n^*$ , the set of original vertices of  $P_n$  forms a dominating set. Hence, the minimum domination number of  $P_n^*$  is  $\gamma(P_n^*) = n$ .

We have,  $\gamma_{d-tc}(P_n^*) = \chi_T(P_n^*) - \gamma(P_n^*) = r + 3 - n$

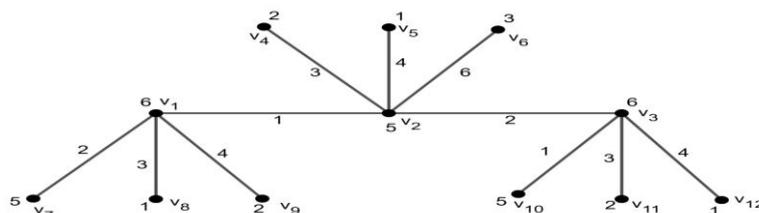
Hence, the Dom-Total chromatic number of Thorny path graph is  $\gamma_{d-tc}(P_n^*) = r + 3 - n$ .

#### Illustration:

Consider the path graph  $P_3$  and let  $r = 3$ . So, attach 3 pendant vertices to each vertex in  $P_3$ .

The total chromatic number of  $P_3^*$  is  $\chi_T(P_3^*) = 6$ . and the minimum domination number is  $\gamma(P_3^*) = 3$

Hence, the Dom-Total chromatic number of  $P_3^*$  is  $\gamma_{d-tc}(P_3^*) = 3$



Total coloring of Thorny path  $P_3^*(r = 3)$  and  $\gamma_{d-tc}(P_3^*) = 3$

### Theorem 3

The Dom-Total chromatic number of thorny cycle graph  $C_n^*$  is  $\gamma_{d-tc}(C_n^*) = r + 3 - n$

**Proof:**

In the cycle graph  $C_n$ , the maximum degree is  $\Delta(C_n) = 2$ .

In the thorny cycle graph  $C_n^*$ , the maximum degree is  $\Delta(C_n^*) = r + 2$

Since the vertex itself must receive a color different from all its incident edges, one additional color is required. So, the total chromatic number of thorny cycle graph is

$$\chi_T(C_n^*) = r + 2 + 1 = r + 3 \quad \forall n \geq 3.$$

To dominate all the vertices of  $C_n^*$ , the set of original vertices of  $C_n$  forms a dominating set.

Hence, the minimum domination number of  $C_n^*$  is  $\gamma(C_n^*) = n$ .

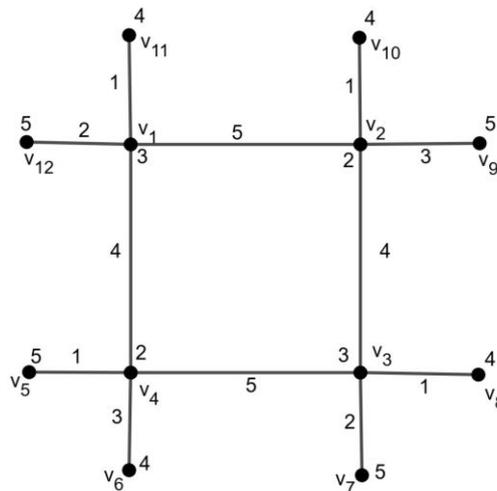
Hence, the Dom-total chromatic number of  $C_n^*$  is  $\gamma_{d-tc}(C_n^*) = r + 3 - n$ .

**Illustration:**

Consider the cycle graph  $C_4$  and let  $r = 2$ . So, attach 2 pendant vertices to each vertex in  $C_4$

Therefore, the total chromatic number of  $C_4^*$  is  $\chi_T(C_4^*) = 5$  and the minimum domination number is  $\gamma(C_4^*) = 4$

Hence, the Dom-Total chromatic number of  $C_4^*$  is  $\gamma_{d-tc}(C_4^*) = 1$



Total coloring of Thorny cycle  $C_4^*$  ( $r = 2$ ) and  $\gamma_{d-tc}(C_4^*) = 1$

**Theorem 4**

The Dom-Total chromatic number of thorny complete graph  $K_n^*$  is  $\gamma_{d-tc}(K_n^*) = r$

**Proof:**

In the complete graph  $K_n$ , the maximum degree is  $\Delta(K_n) = n - 1$

In the thorny Complete graph  $K_n^*$ , each vertex is attached with  $r$  pendant vertices. Hence, the maximum degree becomes  $\Delta(K_n^*) = n - 1 + r$

Since the vertices must receive a color different from all its incident edges, one more color is required. So, the total chromatic number of thorny complete graph is

$$\chi_T(K_n^*) = n - 1 + r + 1 = n + r$$

To dominate all the vertices of  $K_n^*$ , the set of original vertices of  $K_n$  forms a dominating set.

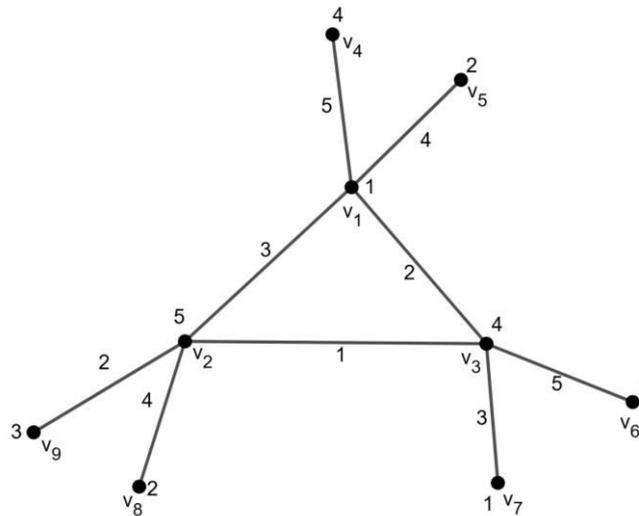
Hence, the minimum domination number of  $K_n^*$  is  $\gamma(K_n^*) = n$

We have,  $\gamma_{d-tc}(K_n^*) = \chi_T(K_n^*) - \gamma(K_n^*) = n + r - n = r$

Hence, the Dom-Total Chromatic number of  $K_n^*$  is  $\gamma_{d-tc}(K_n^*) = r$

**Illustration:**

Consider the complete graph  $K_3$  and let  $r = 2$ . So, attach 2 pendant vertices to each vertex in  $K_3$ . Therefore, the total chromatic number of  $K_3^*$  is  $\chi_T(K_3^*) = 5$  and the minimum domination number is  $\gamma(K_3^*) = 3$ . Hence, the Dom-Total chromatic number of  $K_3^*$  is  $\gamma_{d-tc}(K_3^*) = 2$ .



Total coloring of Thorny complete  $K_3^*$  ( $r = 2$ ) and  $\gamma_{d-tc}(K_3^*) = 2$

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